ON MEASURING THE COMPRESSIBILITY OF SOILS

A. M. Skobev

In [1] a device is described which was used to study the dynamic compressibility of soils. This device in outline constitutes a vertically oriented cup; the sample of the soil is placed on its bottom. The sample is compressed by an elastic piston hit by a freely falling load. In the test process the stress in the sample and the displacement of the end of the piston are registered as functions of time.

The strain is determined simply as the ratio of the displacement to the height of the sample. The process is assumed to be quasi-static.

It is obvious that the process will not always be quasi-static; in particular, immediately after the impact a shock wave can take place in the sample. Consequently, the limits of applicability of the quasi-static analysis must be determined.

When planning a series of tests, it is desirable to know the limits of variation of the stress for a known height of fall of the load and a given elasticity of the piston.

Finally, it is desirable to have a means of verifying, by means of a series of tests, whether some equation of state is satisfied for the soil sample.

These problems will be considered in this note.

1. It is assumed from the very beginning that all quantities depend only on the time t and the single spatial coordinate x. The x axis is directed upward; the ground occupies the interval $[0,l_1]$, and the piston occupies the interval $[l_1,l_1+l_2]$. Further we denote the stress by $\sigma(x,t)$, the strain by $\varepsilon(x,t)$, the velocity by $v(x,t)$, the displacement by $u(x,t)$, the displacement by $\rho(x,t)=\rho_1$ for $0 \le x \le l_1$, and by ρ_2 for $l_1 \le x \le l_1 + l_2$; ρ_1, $\rho_2 = \text{const}$, $\sigma = E_2\varepsilon$ for $l_1 \le x \le l_1 + l_2$; m is the mass of the soil sample per unit surface; v_0 is the modulus of the velocity of the load at the instant of impact. The compressive stress and strain are taken as positive.

In the quasi-static approximation it is assumed that $\partial\sigma/\partial x = 0$, $\partial\varepsilon/\partial x = 0$. For this approximation to be valid, we must stipulate that the stress in all tests be only slightly different from the stress at a certain single point, for example, $x = l_1 + l_2$. Next the weaker condition

$$s_1 \equiv \left|\int_0^t [\sigma(l_1+l_2,\tau)-\sigma(0,\tau)]\,dt\right| \ll s_2 \equiv \left|\int_0^t \sigma(l_1+l_2,\tau)\,d\tau\right| \tag{1.1}$$

is verified.

The equations of motion for the material and the load have the form

$$\rho(x,t)\frac{\partial v(x,t)}{\partial t} = \frac{\partial\sigma(x,t)}{\partial x}, \qquad m\frac{dv(l_1+l_2,t)}{dt} = \sigma(l_1+l_2,t)\,. \tag{1.2}$$

From these equations it follows that

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 11, No. 1, pp. 111-113, January-February, 1970. Original article submitted June 2, 1969.

$$s_1 = \left| \int_0^{l_1+l_2} \rho(x,t)\, v(x,t)\, dx \right|, \qquad s_2 = m\,|v(l_1+l_2,t) - v_0| . \tag{1.3}$$

It is natural to assume that $|v(x,t)| \le v_0$. Then $s_1 < (\rho_1 l_1 + \rho_2 l_2)v_0$ and the condition (1.1) is reduced to the condition

$$\rho_1 l_1 + \rho_2 l_2 \ll m\,[1 - |v(l_1+l_2,t)| / v_0] \tag{1.4}$$

or

$$\rho_1 l_1 + \rho_2 l_2 = \mu m\,[1 - |v(l_1+l_2,t)| / v_0] \quad (\mu \text{ is small}). \tag{1.5}$$

From (1.4), in the first instance, it follows that the parameters of the installation in any case must satisfy the condition

$$\rho_1 l_1 + \rho_2 l_2 \ll m \tag{1.6}$$

i.e., the mass of the load must be much larger than the mass of the soil sample and the piston. In the following we assume this condition as fulfilled.

In the second instance, it is obvious that for $t = 0$,(1.4) is not satisfied (the right side being zero), but as the time elapses, $v(l_1 + l_2,t)$ decreases to zero (the load is slowed down). Consequently, there exists a time interval $[0, t_*]$ over which the process is not quasi-static.

2. We now have to obtain the equations describing the process over the quasi-static segment. In the following we everywhere use the abbreviated notation:

$$-v(l_1,t) = v_1, \quad -v(l_1+l_2,t) - v_1 = v_2$$
$$\varepsilon(l_1,t) = \varepsilon_1, \quad \varepsilon(l_1+l_2,t) = \varepsilon_2, \quad \omega = (ml_1 E_1^{-1} + ml_2 E_2^{-1})^{-1/2} .$$

From quasi-static condition it follows that

$$\varepsilon_1^{\cdot} = v_1 / l_1, \qquad \varepsilon_2^{\cdot} = v_2 / l_2 .$$

Hence the second of Eqs. (1.2) can be written in the form (with $E_2\varepsilon_2 = 0$ taken into account)

$$ml_1\varepsilon_1^{\cdot\cdot} + ml_2E_2^{-1}\sigma^{\cdot\cdot} = -\sigma . \tag{2.1}$$

This equation is closed by the equation of state of the soil (which is not known in advance), and by the boundary conditions

$$\varepsilon(0) = 0, \quad \sigma(0) = 0, \quad l_1\varepsilon_1^{\cdot}(0) + l_2E_2^{-1}\sigma^{\cdot}(0) = v_0 . \tag{2.2}$$

The last one of these conditions is obtained from the definition of $\varepsilon_1^{\cdot}$ and $\varepsilon_2^{\cdot}$, replacing $\varepsilon_2^{\cdot}$ by $E_2^{-1}\sigma^{\cdot}$. Of course, from (2.1), (2.2) we cannot determine $\sigma(t)$ and $\varepsilon(t)$, since the equation of state is not known, but we can make certain estimates.

We assume that the equation of state has the form

$$\frac{d\varepsilon_1}{dt} = \frac{1}{E_1}\frac{d\sigma}{dt} + g(\sigma - \sigma_+(\varepsilon)), \quad g \geqslant 0, \quad g(0) = 0 . \tag{2.3}$$

Substituting (2.3) into (2.1), taking into account the fact that $\omega^2 = m(l_1E_1^{-1} + l_2E_2^{-1})$, we obtain

$$\sigma^{\cdot\cdot} = -\omega^2\sigma - \omega^2 m l_1 g^{\cdot} \tag{2.4}$$

or

$$\sigma = \omega^{-1}\sigma^{\cdot}(0)\sin\omega t - \int_0^t \omega^2 m\, l_1 g^{\cdot} \sin\omega(t-\xi)\, d\xi$$
$$= \omega^{-1}\sigma^{\cdot}(0)\sin\omega t - \omega^3 m\, l_1 \int_0^t \cos\omega(t-\xi)\, g\, d\xi . \tag{2.5}$$

From (2.5) it follows that $\sigma(t) < \omega^{-1}\sigma(0)$ or, with (2.2) and (2.3) taken into account,

$$\sigma_{\max} < v_0 \left(\frac{mE_1 E_2}{l_1 E_2 + l_2 E_1} \right)^{1/2} . \quad (2.6)$$

The time of growth of the load, t_+, also is of importance. For this we have the estimate

$$t_+ < \frac{\pi \sqrt{m}}{2} \left(\frac{l_1 E_2 + l_2 E_1}{E_1 E_2} \right)^{1/2} . \quad (2.7)$$

The inequalities (2.6) and (2.7) allow us to estimate in advance the important quantities $\sigma_{\max}$ and t_+, if we know the quantity E_1 or the so-called dynamic diagram.

From (2.7) we see that the estimate for t_+ does not contain v_0. Apparently this means that the time of growth depends but little on the velocity of the impact.

Finally, we can obtain an estimate for t_*, the instant of time beginning with which the process can be taken as quasi-static.

Let the equation of state have the form $\sigma = E_1\varepsilon$. Then from (2.1), (2.2), and (1.2) we have

$$v = v_0 \cos \omega t, \qquad t_+ = \pi\omega^{-1} / 2 \quad (2.8)$$

and (1.5) gives the following estimates for t_* and t_*/t_+:

$$\mu (1 - \cos \omega t_*) = (\rho_1 l_1 + \rho_2 l_2) / m \quad (2.9)$$

$$t_* / t_+ = 2\pi^{-1} \arccos [1 - (\rho_1 l_1 + \rho_2 l_2) / (m\mu)] . \quad (2.10)$$

Here μ is the same as from (1.5), and characterizes the allowable error. It is natural to stipulate that the condition $t_*/t_+ \ll 1$ be fulfilled, i.e., the time of establishment of a quasi-static state should be small in comparison with the duration of the process. Then from (2.10) we obtain

$$t_* / t_+ = 2\pi^{-1} \sqrt{2 (\rho_1 l_1 + \rho_2 l_2) / (m\mu)}$$

or $m\mu \gg \rho_1 l_1 + \rho_2 l_2$, which is stronger than (1.6).

From (2.10) it is seen that t_*/t_+ does not depend either on v_0 or E_1, i.e., for an elastic medium it does not depend on the equation of state.

Consequently, t_*/t_+ only slightly depends on the state of the sample. Therefore (2.10) can be used for a preliminary estimate of t_*/t_+ in an unknown sample.

3. In order to obtain information about the behavior of the soil from the experimental data we can use various methods. Here we briefly consider two of them.

The first of them consists of setting up a family of curves (σ,ε) for constants $\varepsilon^\cdot$ and assuming that this in certain cases can replace the exact equation.

Later, by way of an example of an elastic sample, it will be shown that device described above is not suitable for this. The condition of applicability of the first method can be written in the form

$$| \varepsilon^{\cdot\cdot} t_+ | \ll | \varepsilon^\cdot | \quad \text{or} \quad | \varepsilon^{\cdot\cdot} t_+ / \varepsilon^\cdot | < \mu_1 \quad (3.1)$$

where μ is sufficiently small. This simply means that $\varepsilon^\cdot$ must not significantly vary over the characteristic time of the process. Furthermore, it is reasonable to choose the criterion of accuracy μ_1 as equal to μ.

For an elastic sample ($\sigma = E_1\varepsilon_1$) the condition (3.1) is transformed into $\frac{1}{2}\pi \operatorname{tg} \omega t < \mu$, i.e., (3.1) is satisfied only for $t < t_0$, where $t_0 = \omega^{-1} \operatorname{arc\,tg} (2\mu/\pi)$.

Thus, $\varepsilon^\cdot$ can be taken as constant only for $t_* < t < t_0$, i.e., we must have $t_0 > t_*$. If (2.9) is fulfilled, then this inequality can be rewritten in the form (t_* must be small)

$$\mu^3 > \frac{\pi^2}{2} \frac{\rho_1 l_1 + \rho_2 l_2}{m} .$$

This is a very strong inequality.

The second method consists of assuming a certain equation of state of the soil, and a series of tests is set up to verify this assumption.

For example, we assume that the soil sample is described by Eq. (2.3) with linear g and σ_+. Let $\sigma_*(t)$ and $\varepsilon_*(t)$ be the solution of the system (2.1)-(2.3) with $v_0 = v_*$. Then the solution with an arbitrary v_0 has the form $\sigma(t) = (v_0/v_*)\,\sigma_*(t)$, $\varepsilon(t) = (v_0/v_*)\,\varepsilon_*(t)$. Thus a series of tests with different v_0 allows us to decide whether or not the given assumption is applicable.

Concluding, the author thanks the participants of the seminar of the Section of Dynamics of Nonelastic Media of the Institute for Problems of Mechanics of the Academy of Sciences of the USSR for the discussion of this paper.

LITERATURE CITED

1. V. V. Mal'nikov and G. V. Rykov, "Effect of rate of deformation on the compressibility of loess soils," PMTF [Journal of Applied Mechanics and Technical Physics], no. 2, 1965.